TABLE III
ENERGY CONSERVATION FOR $\mathbf{m}$ (STATIC CASE)

Method of calculation	Calculated energy
$\frac{1}{2}\mu_o \iiint_V H^2(T)\, dV$	102.2655 pJ
$\int_0^T \iint_S \mathbf{E}(t) \times \mathbf{H}(t) \cdot d\mathbf{S}\, dt$	0.0041 pJ
$\int_0^T V_{loop}(t) I_s(t)\, dt$	102.2695 pJ

TABLE IV
POWER BALANCE FOR $\mathbf{p}$ AT 1.8 GHz. $I_{\rm gap} = 1$ A

Method of calculation	Calculated power
$\frac{1}{12\pi}\sqrt{\frac{\mu_o}{\varepsilon_o}}\left(\frac{\omega\Delta}{c}\right)^2 \left\lvert I_{gap}(\omega)\right\rvert^2$	88.88 mW$^{(a)}$
$\frac{1}{12}\mathrm{Re}\iint \mathbf{E}_\omega \times \mathbf{H}^*_\omega \cdot d\mathbf{S}$	88.97 mW$^{(b)}$ 88.83 mW$^{(c)}$
$\frac{1}{2}\mathrm{Re}\left[V_{gap}(\omega) I^*_{gap}(\omega)\right]$	88.94 mW$^{(b)}$ 88.02 mW$^{(c)}$

(a) analytic result. *(b)* resistor. *(c)* zero mean value pulse.

used instead of a lumped resistor). Table III shows the same balance for $\mathbf{m}$. Here, $V_{\rm loop}(t)$, calculated using E_ϕ, is the EMF along the current loop. Finally, Table IV shows, for $\mathbf{p}$, a balance in the frequency domain (1.8 GHz) between the source power and corresponding flux leaving the grid boundaries. To avoid static fields, both methods described above are used. Calculated data are compared to the analytic value of the power delivered by an ICE of the same length (first row). The observed deviation is within 0.15%; a figure that would not be reached without using methods to prevent static fields. If these fields are not removed in accordance with the techniques suggested here, special post-processing of data [9], [10] is needed. Otherwise, an instability is produced by Fourier transforming, even when time-domain fields' values converge satisfactorily.

V. CONCLUSIONS

Apart from high-frequency applications, the FDTD method is also capable of solving static problems. The nonrecognition of this ability has led some investigators to consider such solutions, which may cause serious problems in the frequency domain, as numerical artifacts. In this paper, besides proposing techniques to eliminate them, we have presented the possibility of using the FDTD method in static and quasi-static calculations (electric and magnetic dipoles). However, it has not yet been determined if the FDTD method can efficiently compete with consolidated scalar techniques because it typically still involves an excessive number of time steps needed to attain acceptable solutions.

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An Improved Algorithm of Constructing Potentials From Cauchy Data and Its Application in Synthesis of Nonuniform Transmission Lines

Gaobiao Xiao and Ken'ichiro Yashiro

***Abstract*—It is required to construct a potential function from Cauchy data in the synthesis of arbitrarily terminated nonuniform transmission lines. An improved algorithm for this problem is discussed in this paper. With the proposed algorithm, not only is computation time reduced, but the possible divergence of the potential function that sometimes occur when adopting the successive approximation method is also avoided. It has been applied successfully to designs of nonuniform transmission-line filters or tapers through solving inverse problems.**

***Index Terms*—Filter, inverse problem, nonuniform transmission lines.**

I. INTRODUCTION

The synthesis of nonuniform transmission lines (NTLs) has been investigated by many authors [1]–[4] and, among them, the methods based on solving inverse scattering problems are probably most promising. We have proposed a numerical method to synthesize arbitrarily terminated NTLs by solving a related inverse classic

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G. Xiao is with the Graduate School of Science and Technology, Chiba University, Chiba City 263-8522, Japan and also with the Faculty of Electronics and Information, Hunan University, Hunan 410082, China.
K. Yashiro is with the Faculty of Engineering, Chiba University, Chiba City 263-8522, Japan.
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Sturm–Liouville problem, which may be used to design NTL filters or tapers [5]–[7].

In the method proposed in [5], the telegrapher's equation that describing the voltage $v(x, t)$ and current $i(x, t)$ on an NTL is cast into a classical Sturm–Liouville equation

$$-\phi''(x, \lambda) + q(x)\phi(x, \lambda) = \lambda\phi(x, \lambda) \tag{1}$$

where $\phi(x, \lambda) = v(x, \lambda)/\sqrt{Z_0(x)}$, λ is the angular frequency, $Z_0(x)$ is the characteristic impedance of the NTL, and x is the electric position defined in [1]. The potential function $q(x)$ relates to $Z_0(x)$ by $[1/\sqrt{Z(x)}]'' - q(x)[1/Z(x)] = 0$.

We may synthesize an NTL in two steps. At first, approximate a requisite S-parameter in terms of entire functions by adjusting eigenvalue sequences corresponding to (1), and then solve the inverse problem of recovering $q(x)$ from those eigenvalue sequences. The first step may be carried out by solving a constraint approximation problem, while the inverse problem in the second step may be solved by a numerical method, as described in [8].

Denote two linearly independent solutions of (1) by $y_1(x, \lambda)$ and $y_2(x, \lambda)$. They are subject to boundary conditions of $y_1(0, \lambda) = 1$, $y_1'(0, \lambda) = 0$, and $y_2(0, \lambda) = 0$, $y_2'(0, \lambda) = 1$. Theoretically, $q(x)$ can be recovered from the two zero sequences of $y_2(\ell, \lambda)$ and $y_2'(\ell, \lambda)$. The following integral transformation [8]–[10] is well used:

$$y_2(x, \lambda) = \frac{\sin\left(\sqrt{\lambda}x\right)}{\sqrt{\lambda}} + \int_0^x K(x, s)\frac{\sin\left(\sqrt{\lambda}s\right)}{\sqrt{\lambda}}\, ds. \tag{2}$$

The kernel $K(x, t)$ satisfies

$$K_{tt}(x, t) - K_{xx}(x, t) + q(x)K(x, t) = 0 \tag{3}$$

$$K(x, x) = \frac{1}{2}\int_0^x q(s)ds$$
$$K(x, 0) = 0. \tag{4}$$

Integrating (3) by parts in a proper region yields the following Volterra-type equation [8]:

$$q(x) = 2\Big[K_t(\ell, 2x - \ell) + K_x(\ell, 2x - \ell)\Big] - 2\int_x^\ell q(s)K(s, 2x - s)\, ds. \tag{5}$$

$K(\ell, 2x - \ell)$ and $K_x(\ell, 2x - \ell)$ can be calculated from λ_i and μ_i, and are usually called Cauchy data [8]. With these Cauchy data, we can solve (5) by the successive approximation method. A typical algorithm of the successive approximation method may be organized as follows: assume initial data $q(x) = 0$ for the first iteration and solve the kernel $K(x, t)$ from (3) in region $| t |\leq x \leq \ell$, then from (5), calculate a renewed data of $q(x)$; use the renewed data of $q(x)$ as initial data for the next iteration and repeat the above calculation. The process is stopped when a stable $q(x)$ is reached. In the case of an NTL filter, $q(x)$ is usually large and also oscillates rapidly. We found that about 15 iterations are often needed to get a stable numerical solution of $q(x)$. Moreover, we even cannot obtain a stable numerical solution of $q(x)$ with the successive approximation method when the fractional bandwidth of the NTL filter is very wide. This is possibly caused by the fact that the first guessed data of $q_0(x) = 0$ may be far away from the real value of $q(x)$. In this paper, we present an algorithm to find the numerical solutions without iterations.

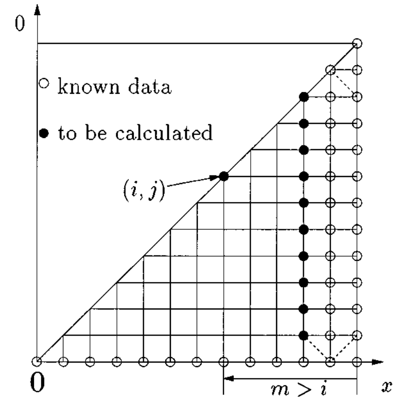

Fig. 1. Mesh structure for calculating $K(x, t)$ and $q(x)$.

II. Improved Algorithm

Let $x = i\Delta$, $t = j\Delta$, where $\Delta = \ell/N$, $i = 0, 1, \ldots, N$, $j = 0, 1, \ldots, i$. Here, we use the following modified formula to discretize (5):

$$q(i) = 2\Big[K_t(N, 2i - N) + K_x(N, 2i - N)\Big] - 2\Delta\sum_{m=i+1}^{N} q(m)K(m, 2i - m) \tag{6}$$

and from (3), we can write

$$K(i, j) = K(i + 1, j + 1) + K(i + 1, j - 1) - K(i + 2, j) + \Delta^2 q(i + 1)K(i + 1, j). \tag{7}$$

In this way, $q(i)$ and $K(i, j)$ are only related to data in region $m > i$. In other words, if those data in region $m > i$ are known, then $q(i)$ and $K(i, j)$ can be calculated directly.

In Fig. 1, the values at points $\circ$ are all known and serve as initial data, namely, $K(N, j)$, $K(N - 1, j)$, $j = 0, \ldots, N$ and $K(i, 0) = 0$, $i = 0, \ldots, N$.

If we examine (5) more closely, we will see that at $x = \ell$, $q(\ell) = 2[K_t(\ell, \ell) + K_x(\ell, \ell)]$ is actually the exact value of $q(x)$ at $x = \ell$. Certainly, we should not assume that $q(\ell) = 0$ at first and then find the already known data for $q(\ell)$ through iterations, as in the algorithm described in [8]. Thus, in this paper, initial data also include $q(N) = 2[K_t(N, N) + K_x(N, N)]$, and $q(N - 1) = 2[K_t(N, N - 2) + K_x(N, N - 2)] - 2\Delta q(N)K(N, N - 2)$.

Basically, by using (6) and (7) alternatively, we can find $q(N - 2)$ and $K(N - 2, j)$ from known data of $K(N, j)$, $K(N - 1, j)$ and $q(N)$, $q(N - 1)$. The resultant values are already those we can expect to obtain for a specified spacing Δ, and need not be recalculated through iterations. In the same way, we can calculate the values of $K(i, j)$ and $q(i)$ at line $i = N - 3$, and so on, until all the values of $K(i, j)$ and $q(i)$ in region $|t| \leq x \leq \ell$ are found. No iteration is needed, and stable numerical solutions of $q(x)$ are always obtainable.

The memory needed in the present method is also much smaller. In the successive approximation method, all the values of the kernel, an $N \times N$ matrix, should be stored for the next iteration, while only three successive lines of the kernel data, a $3 \times N$ matrix, need be stored in the new method.

III. Example

Take an NTL bandpass filter with 100% fractional bandwidth as an example. The NTL filter is terminated with Z_ℓ at $x = \ell$ and Z_g at $x = 0$. The incident and reflected waves at $x = 0$ serve as the input

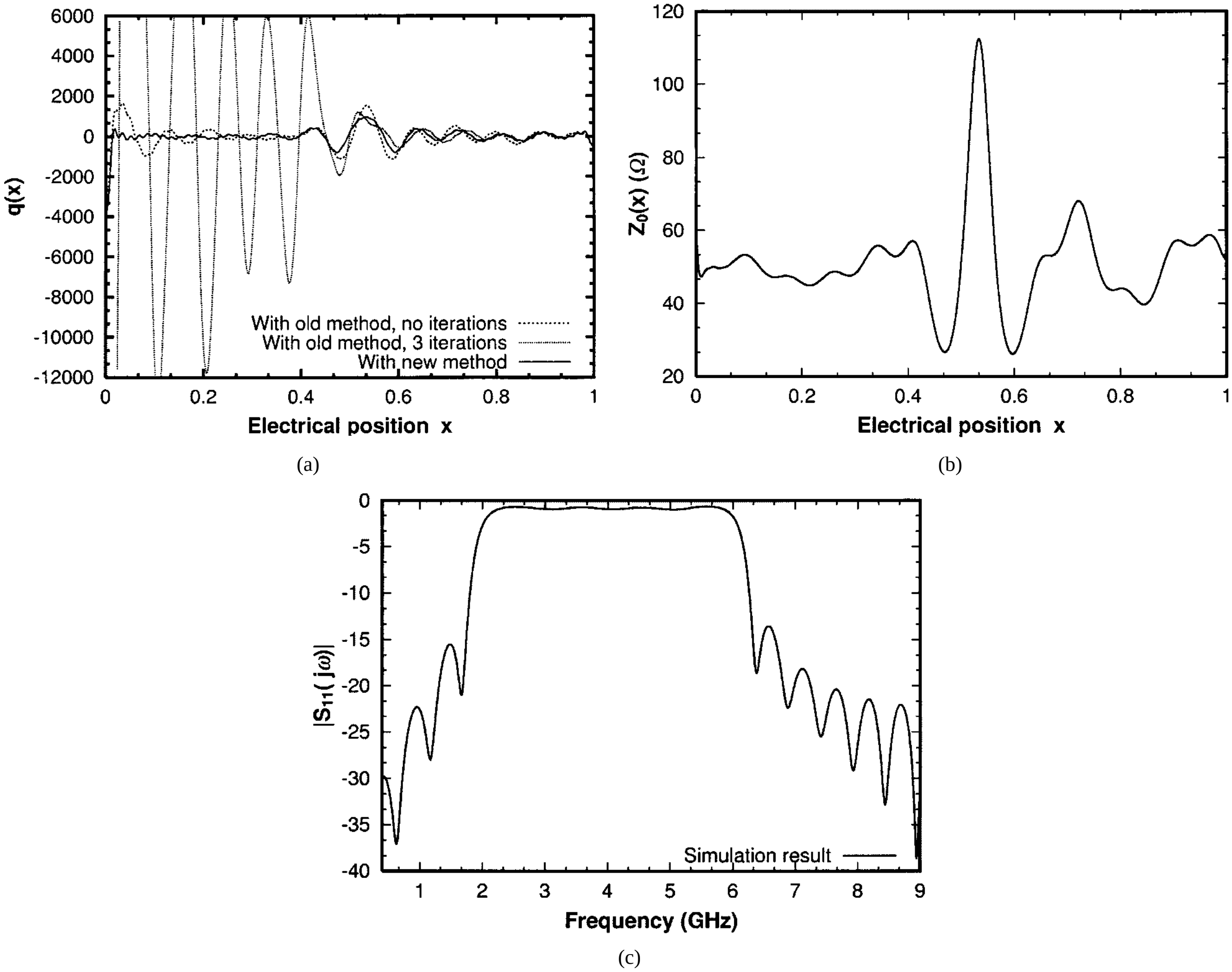

Fig. 2. Results of an NTL filter with a passband of 2–6 GHz. (a) Constructed potentials. Convergent $q(x)$ is obtained by the proposed method (new method), while $q(x)$ diverges by the successive approximation method (old method). $N = 10\,000$. (b) Corresponding characteristic impedance of the designed NTL filter. (c) Simulated $|S_{11}(j\omega)|$ of the NTL filter.

signal and output signal of the filter, respectively. Thus, the frequency response of the filter may be characterized by S_{11}. It is assumed that $|S_{11}| = 0.9$ through the passband of 2–6 GHz, and $|S_{11}| = 0$ elsewhere.

Here, we choose the normalizing frequency $f_c = 1$ GHz, and $\ell = 1$. This means that the length of the NTL filter is the same as the guided wavelength at f_c [7]. When time delay is taken into account, the requisite $S_{11}(j\omega)$ may be written as $S_{11}(j\omega) = 0.9e^{-j\omega}$ for $\omega \in (4\pi,\, 12\pi)$. Following the algorithm described in [5] and assuming that $k(1) = 0$, we use the following equations to approximate $S_{11}(j\omega)$:

$$y_2(1,\,\lambda) = \frac{1}{\omega}\sqrt{\frac{Z_g Z_\ell}{Z_0(0)Z_0(1)}}\,\Im\left(\frac{1+S_{11}}{S_{12}}\right) \tag{8}$$

$$y_2'(1,\,\lambda) = \sqrt{\frac{Z_g Z_0(1)}{Z_0(0)Z_\ell}}\,\Re\left(\frac{1+S_{11}}{S_{12}}\right) \tag{9}$$

where the entire functions $y_2(1,\,\lambda)$ and $y_2'(1,\,\lambda)$ can be expressed as

$$y_2(1,\,\lambda) = \frac{\sin\left(\sqrt{\lambda}\right)}{\sqrt{\lambda}}\prod_{j=1}^{M}\frac{\mu_j - \lambda}{Q_j - \lambda} \tag{10}$$

$$y_2'(1,\,\lambda) = \cos\left(\sqrt{\lambda}\right)\prod_{j=1}^{M}\frac{\lambda_j - \lambda}{P_j - \lambda}. \tag{11}$$

μ_j and λ_j are zeros of $y_2(1,\,\lambda)$ and $y_2'(1,\,\lambda)$, respectively. $\sqrt{P_j} = (j-0.5)\pi$, $\sqrt{Q_j} = j\pi$. and M is the number of zeros taken into account. In the example, we choose $M = 60$, thus, the frequency range under consideration is up to $\omega = 60\pi$, which is enough to cover the passband. From the facts that μ_j and λ_j interlace and $y_2(1,\,\lambda_1) > 0$, $y_2'(1,\,\mu_1) < 0$, we have to find μ_j, λ_j and $Z(0)$, $Z(1)$, which satisfy the following equations:

$$y_2(1,\,\lambda_i) = (-1)^{i+1}\sqrt{\frac{Z_g Z_\ell}{Z(0)Z(1)}}\,\frac{g(\lambda_i)}{\sqrt{\lambda_i}} \tag{12}$$

$$y_2'(1,\,\mu_i) = (-1)^{i}\sqrt{\frac{Z_g Z(1)}{Z(0)Z_\ell}}\,g(\mu_i), \qquad i = 1,\,\ldots,\,M \tag{13}$$

where $g(\lambda) = |1 + S_{11}/S_{21}|$. Equations (12) and (13) correspond to [5, eqs. (67) and (68)] (but notice that there is a misprint concerning the signs of [5, (67) and (68)]). In addition, at $\lambda = 0$, we have $y_2'(1,\,0) = \sqrt{Z(1)/Z(0)}$, which is a condition discussed in [7], and is equivalent to the condition of $G_2(0) = B_2 g(0)$, which has been adopted in [5].

Equations (12) and (13) are nonlinear equations and can be solved approximately by minimizing the following object function through adjusting μ_j, λ_j and $Z(0)$, $Z(1)$:

$$\begin{aligned} E_R[\lambda_i, \mu_i, Z(0), Z(1)] &= \sum_{i=1}^{M} \left[\frac{\sin\left(\sqrt{\lambda_i}\right)}{\sqrt{\lambda_i}} \prod_{j=1}^{M} \frac{\mu_j - \lambda_i}{Q_j - \lambda_i} - (-1)^{i+1} \sqrt{\frac{Z_g Z_\ell}{Z(0)Z(1)}} \frac{g(\lambda_i)}{\sqrt{\lambda_i}} \right]^2 \\ &+ \sum_{i=1}^{M} \left[\cos\left(\sqrt{\mu_i}\right) \prod_{j=1}^{M} \frac{\lambda_j - \mu_i}{P_j - \mu_i} - (-1)^i \sqrt{\frac{Z_g Z(1)}{Z(0) Z_\ell}} g(\mu_i) \right]^2 \\ &+ \left[\prod_{j=1}^{M} \frac{\lambda_j}{P_j} - \sqrt{\frac{Z(1)}{Z(0)}} \right]^2. \end{aligned} \tag{14}$$

The resultant μ_j and λ_j are used to generate Cauchy data, while the resultant $Z(0)$ and $Z(1)$ are used as boundary data to calculate $Z(x)$ from $q(x)$. The successive approximation method described in [8] failed to recover $q(x)$ from the Cauchy data. As is shown in Fig. 2(a), $q(x)$ diverges after two iterations. However, we can use the proposed method to calculate $q(x)$ without difficulty. For reference, Fig. 2(b) and (c) shows the related characteristic impedance and simulated $|S_{11}(j\omega)|$ of the NTL filter, respectively.

IV. Conclusions

When applied to constructing $q(x)$ from Cauchy data, the successive approximation method suffers from two disadvantages, i.e., several iterations are needed and they sometimes fail when the corresponding $q(x)$ is large and oscillates rapidly. In the present algorithm, $q(x)$ can be recovered straightforwardly from Cauchy data by using the exact data of $q(x)$ at $x = \ell$ and adopting a proper discrete form of the related integral equation; hence, the above disadvantages are avoided. The numerical error of the proposed method mainly depends upon sampling spacing Δ.

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The Reentrant Wide-Band Directional Filter

Anatoly Petrovich Gorbachev

***Abstract*—A new type of a wide-band microwave filter is described and named the reentrant directional filter, in which resonance occurs in the form of a traveling wave rather than in the conventional form of a standing wave. This device is the network, which has the constant input impedance and is manufactured as the directional coupler's free construction. An analysis of the reentrant directional filter shows it to have advantages in the case of wide-bands when compared to previously used directional filters. This filter finds application in multiplexers, as well as in matched bandpass (band-stop) filters by using planar multilayer transmission-line technology. Experimental results verify the theoretical approach.**

***Index Terms*—Directional filters, microwaves, multilayer, strip-line components, wide-band.**

I. Introduction

It is well known [1] that directional filters are one type of device capable of performing systems that use frequency-division multiplexing. For example, wide-band multiplexers have been analyzed and constructed as described in [2].

The basic directional filter proposed in [3] has been developed in many papers [4]–[6]. However, the above-mentioned directional filters are themselves badly adapted to multilayer (multilevel)-strip transmission-line technology, which has been an increased interest in recent years in RF integrated circuits. For instance, the analysis and design of multilayer coupled-line directional couplers has recently been reported in [7].

This paper present a novel wide-band directional filter called a "reentrant directional filter," which is better adapted itself to multilayer planar transmission-line technology through the full screening of its fragments.

II. Analysis

To better understand the above-mentioned directional filter, it should be analyzed on a base of a strip-coaxial model, although practical devices remain to be constructed in multilayer-strip transmission-line realization.

Fig. 1 shows the new reentrant directional filter (patent pending). Conductors A–D are coaxial-line center conductors of characteristic impedance Z_B and electrical length θ_B with relative dielectric constant ε_{rB} of the coaxial-line medium (brief expression—"transmission line (Z_B, θ_B)") within the loop L of electrical length $4\theta_N$. The loop L is formed from the center conductor of the transmission line of characteristic impedance Z_N with corresponding dimensions w_a, w_B, t, and b and relative dielectric constant ε_{rN} of the directional filter medium within ground conductor G. The arms of the conductors A–D are connected together by a narrow conducting link. The link is designed to be narrow so that there will be no propagation around the end of the center conductors, but rather, that the even and odd excitation will be terminated by open and short circuits, respectively. Distance a is chosen to avoid electromagnetic coupling between opposite edges of loop conductor L. The external arms of the conductors A–D are directional filter terminal ports 1–4, respectively.

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The author is with the Department of Radiophysics, Novosibirsk State Technical University, Novosibirsk 630092, Russia.

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